Robust Control of Solid Oxide Fuel Cell
Ultra-Capacitor Hybrid System

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Abstract

Mitigating fuel starvation and improving load-following capability of solid oxide fuel cells (SOFC) are conflicting control objectives. In this paper we address this issue using a hybrid SOFC ultra-capacitor configuration. Fuel starvation is prevented by regulating the fuel cell current using a steady-state invariant relationship involving fuel utilization, fuel flow and current. Two comprehensive control strategies are developed. The first is a Lyapunov-based nonlinear control and the second is a standard $H_\infty$ robust control. Both strategies additionally control the state of charge (SOC) of the ultra-capacitor that provides transient power compensation. A hardware-in-the-loop test-stand is developed where the proposed control strategies are verified.
1 INTRODUCTION

Among different fuel cell technologies, SOFC technology has attracted significant interest in recent years. SOFCs are solid state devices that are fuel flexible, tolerant to impurities and operate at high temperatures (800° – 1000°C). The high temperatures allow internal reforming, promote rapid reaction kinetics with non-precious metals and produce high quality by-product heat for co-generation or for use in a bottoming cycle, [1, 2].

In spite of these attributes, application of SOFCs has been limited due to their poor load following capability [3], and has particularly precluded their use in applications involving rapid power variations. This is a common drawback of fuel cells. It is attributed to the slow dynamic response of the fuel and air delivery systems consisting of valves, pumps and reformers, [4, 5, 6, 7]. It is manifested as hydrogen or oxygen starvation and drastic voltage drop when the fuel cell is directly exposed to rapid power transients. The phenomenon adversely affects cell durability through anode oxidation [7] and reversal of cell potential, leading to catalyst corrosion [8]. Multiple authors have addressed this issue by augmenting the fuel cell with an electrical storage device. In [4], the authors develop a current control strategy to minimize the fuel cell’s voltage droop. In [5], fuel cell current is rate limited to prevent starvation in a fuel cell ultra-capacitor hybrid. In [9], the authors propose a robust load governor for preventing oxygen starvation. In [6], a Model Predictive Control (MPC) is developed for a fuel cell ultra-capacitor system that minimizes oxygen starvation, bounds the ultra-capacitor’s state-of-charge (SOC), and prevents compressor surge and choke. An MPC based approach for improving battery performance and avoiding fuel cell and battery degradation is used in [10].

In addition, a number of papers have proposed control of hybrid fuel cells without specifically addressing the above mentioned constraints, but providing a rich spectrum of control approaches. In [11, 12, 13], rule-based control strategies are developed where the hybrid system switches between discrete operating modes. In [14], the proposed control strategy regulates the fuel cell current based on the battery SOC. In [15], a two-loop control strategy
is proposed for a hybrid fuel cell where an inner loop regulates the DC bus voltage and an outer loop regulates the fuel cell current. In [16], a sliding-mode control [17] is developed for a fuel cell hybrid system. In [18], the authors develop a differential flatness [19] based control for a fuel cell ultra-capacitor system. In [20], optimal power distribution is obtained in a hybrid fuel cell vehicle by local minimization of an equivalent fuel consumption variable.

A majority of the work mentioned above pertain to PEMFCs. Control development for hybrid SOFCs systems are in preliminary stages. For SOFCs, the load-following limitations of the fuel cell are reflected in the transient response of a performance variable, namely fuel utilization. Fuel utilization $U$ is defined as the ratio of hydrogen consumption to the net available hydrogen in the anode. In the formulation of $U$, not only is the available hydrogen considered but the hydrogen that can be generated from other species through internal reforming are also accounted for [21]. This is because of SOFC’s tolerance to impurities that allows the reformer exhaust gas-mixture to be directly sent to the anode with minimal or no purification. While high utilization implies high efficiency, very high utilization leads to reduced partial pressure of hydrogen in the anode, leading to voltage drop and anode oxidation [7]. Low fuel utilization on the other hand results in under-used fuel, [22]. Typically, 80 – 90% is set as the target range for maximizing the cell’s electric efficiency, [21, 22, 23].

Control of $U$ around a target value has been adopted in a number of studies [24, 25, 26, 27]. However, measurement of $U$ requires several species-specific concentration sensors that are avoided due to cost and reliability considerations. Observer designs are possible [28, 29, 30, 31], however they can be computation intensive, and rely on accurate mathematical models. In contrast, in this paper we propose control of $U$ by combining an invariant relationship of the SOFC system [32], with a current regulation strategy. The approach achieves a target steady-state $U$ and attenuates transient departure from this target. The deficit or surplus power delivered by the SOFC due to current regulation is compensated by an ultra-capacitor. Current regulation is incorporated within a robust control scheme that additionally controls the ultra-capacitor’s SOC. The control objectives are satisfied in
presence of uncertainties. Two control strategies are developed, a nonlinear control and an $H_{\infty}$ control. This work addresses hydrogen starvation but oxygen starvation is not considered as it is seldom observed in typical SOFC systems. This is because in the absence of coolants, excess air (air utilization $\approx 20 - 25\%$, [33]) is used for temperature control [7].

SOFC systems are in developmental stages and their commercial availability is extremely limited. Also, the costs and necessary infrastructural support for experimentation with prototypes are prohibitive. Hence, for control verification we use a control-oriented mathematical model of a tubular SOFC system developed in our prior research, presented in [32, 34]. The model captures the thermodynamics, chemical kinetics, heat transfer and pressure dynamics phenomena and has been validated against results in [7] and [33]. We develop an experimental test-stand where the SOFC system is emulated by combining a real-time implementation of the model with a programmable power supply. The emulated SOFC is integrated with commercially available power electronics components and an ultra-capacitor.

The paper is organized as follows: A system description and open-loop results are presented in section 2. The invariant relationship is derived and used for open-loop control in section 3. The current regulation method is presented in section 4, followed by a discussion in section 5 on system-induced flow delays. The hybrid fuel cell configuration and the control designs are presented in sections 6 and 7. The experimental test-stand is explained in section 8, followed by results in section 9. Finally concluding remarks and acknowledgments are stated in sections 10 and 11, and references are listed.

2 FUEL CELL SYSTEM

We consider a steam reformer based tubular SOFC system. The system consists of three primary components, namely, the steam reformer, the fuel cell stack and the combustor. Methane is chosen as the fuel for the system, with a molar flow rate of $\dot{N}_f$. The analysis and control approach presented in this paper can be extended to other fuels and system
configurations. The system is presented in Fig.1.

The reformer produces a hydrogen-rich gas which is supplied to the anode of the fuel cell. Electrochemical reactions occurring at the anode due to current draw results in a steam-rich gas mixture at its exit. A known fraction $k$ of the anode exhaust is recirculated through the reformer into a mixing chamber where fuel is added. The mixing of the two fluid streams and pressurization is achieved in the gas mixer using an ejector or a recirculating fuel pump, [35]. The steam reforming process occurring in the reformer catalyst bed is an endothermic process. The energy required to sustain the process is supplied from two sources, namely, the combustor exhaust that is passed through the reformer, and the aforementioned recirculated anode flow, as shown in Fig.1. The remaining anode exhaust is mixed with the cathode exhaust in the combustion chamber. The combustor also serves to preheat the cathode air which has a molar flow rate of $\dot{N}_{\text{air}}$. The tubular construction of each cell causes the air to first enter the cell through the air supply tube and then reverse its direction to enter the cathode chamber. For steam reforming of methane we consider a packed-bed tubular reformer with nickel-alumina catalyst. The three main reactions in steam reforming of methane are, [36]:

\begin{align}
(I) \quad CH_4 + H_2O & \leftrightarrow CO + 3H_2 \\
(II) \quad CO + H_2O & \leftrightarrow CO_2 + H_2 \\
(III) \quad CH_4 + 2H_2O & \leftrightarrow CO_2 + 4H_2
\end{align}

Internal reforming reactions I, II and III in Eq.(1) occur in the anode due to high tempera-
tures and the presence of nickel catalyst. The primary electrochemical process is

\[(\text{IV}) \quad H_2 + O^2- \rightarrow H_2O + 2e \quad (2)\]

Simultaneous electrochemical conversion of CO to CO$_2$ is ignored since its rate is much slower in presence of reactions II and IV, [37]. Details of the system model are presented in [34]. Fuel utilization $U$ is mathematically defined as, [7, 21, 23]:

\[
U \triangleq 1 - \frac{\dot{N}_o (4X_{1,a} + X_{2,a} + X_{4,a})}{\dot{N}_in (4X_{1,r} + X_{2,r} + X_{4,r})} \quad (3)
\]

where, $X_{1,a}$, $X_{2,a}$, $X_{4,a}$ and $X_{1,r}$, $X_{2,r}$, $X_{4,r}$ are the molar concentrations of CH$_4$, CO and H$_2$ in the anode and the reformer respectively and $\dot{N}_o$ and $\dot{N}_in$ are shown in Fig.1. Eq.(3) is based on the observation that a CH$_4$ and a CO molecule can yield at most four molecules and one molecule of H$_2$ respectively, as indicated by reactions I, II and III in Eq.(2).

## 3 OPEN-LOOP CONTROL OF $U$

The molar balance equations of individual species in the reformer and anode are:

\[
\begin{align*}
\dot{N}_rX_{1,r} + N_r\dot{X}_{1,r} &= k\dot{N}_o X_{1,a} - \dot{N}_in X_{1,r} + \mathcal{R}_{1,r} + \dot{N}_f \\
\dot{N}_rX_{2,r} + N_r\dot{X}_{2,r} &= k\dot{N}_o X_{2,a} - \dot{N}_in X_{2,r} + \mathcal{R}_{2,r} \\
\dot{N}_rX_{3,r} + N_r\dot{X}_{3,r} &= k\dot{N}_o X_{3,a} - \dot{N}_in X_{3,r} - \mathcal{R}_{1,r} - \mathcal{R}_{2,r} \\
\dot{N}_rX_{4,r} + N_r\dot{X}_{4,r} &= k\dot{N}_o X_{4,a} - \dot{N}_in X_{4,r} - 4\mathcal{R}_{1,r} - \mathcal{R}_{2,r} \\
\dot{N}_rX_{5,r} + N_r\dot{X}_{5,r} &= k\dot{N}_o X_{5,a} - \dot{N}_in X_{5,r} + 2\mathcal{R}_{1,r} + \mathcal{R}_{2,r}
\end{align*}
\]

\[
\begin{align*}
\dot{N}_aX_{1,a} + N_a\dot{X}_{1,a} &= \dot{N}_in X_{1,r} - \dot{N}_o X_{1,a} + \mathcal{R}_{1,a} \\
\dot{N}_aX_{2,a} + N_a\dot{X}_{2,a} &= \dot{N}_in X_{2,r} - \dot{N}_o X_{2,a} + \mathcal{R}_{2,a} \\
\dot{N}_aX_{3,a} + N_a\dot{X}_{3,a} &= \dot{N}_in X_{3,r} - \dot{N}_o X_{3,a} - \mathcal{R}_{1,a} - \mathcal{R}_{2,a} \\
\dot{N}_aX_{4,a} + N_a\dot{X}_{4,a} &= \dot{N}_in X_{4,r} - \dot{N}_o X_{4,a} - 4\mathcal{R}_{1,a} - \mathcal{R}_{2,a} - r_e \\
\dot{N}_aX_{5,a} + N_a\dot{X}_{5,a} &= \dot{N}_in X_{5,r} - \dot{N}_o X_{5,a} + 2\mathcal{R}_{1,a} + \mathcal{R}_{2,a} + r_e
\end{align*}
\]
where, \( r_e \) is the rate of electrochemical reaction. In Eqs. (4) and (5), \( X_{i,r} \) and \( X_{i,a} \) are the molar concentrations of species in the reformer and anode respectively, with \( i = 1, 2, \cdots, 5 \) representing \( CH_4, CO, CO_2, H_2 \) and \( H_2O \) in that order. \( N_r \) and \( N_a \) are the molar contents of the reformer and the anode, and \( k \) is the constant and known recirculation fraction shown in Fig.1. \( R_{1,r}, R_{2,r} \) and \( R_{1,a}, R_{2,a} \) are the rates of formation of \( CH_4 \) and \( CO \) in the reformer and anode respectively. In Eq. (6), \( i_{fc} \) is the fuel cell current, \( N_{cell} \) is number of series-connected cells, \( n = 2 \) is the number of electrons participating in an electrochemical reaction, and \( F = 96485.34 \text{Coul./mole} \) is the Faraday’s constant. Further details about the equations can be found in \([32, 34]\). From Eqs. (3), (4), (5) and (6), noting that the left hand sides of Eqs. (4) and (5) are zero at steady-state, the steady-state utilization \( U_{ss} \) is obtained as

\[
U_{ss} = \frac{1 - k}{\left( 4nFN_f/i_{fc}N_{cell} \right) - k}
\]

Note that Eq. (7) is independent of the reaction rates \( R_{1,r}, R_{2,r}, R_{1,a}, R_{2,a} \) and the flow rates \( \dot{N}_{in}, \dot{N}_o \). Equation (7) is valid in steady-state and is invariant with respect to variations in operating temperature, operating pressure, mass of reforming catalyst, air flow rate and operating Steam-to-Carbon ratio (STCR) \( \text{STCR} \triangleq k\dot{N}_oX_{5,a}/\left[ \dot{N}_f + k\dot{N}_oX_{1,a} + k\dot{N}_oX_{2,a} \right], [34] \). Thus, Eq. (7) represents an invariant relationship between steady-state fuel utilization \( U_{ss} \), fuel cell current \( i_{fc} \), and fuel flow rate \( \dot{N}_f \). Given a target \( U_{ss} \), it can be used to determine \( \dot{N}_f \) if \( i_{fc} \) is known and vice-versa. The invariance is attributed to the definition of \( U \) in Eq. (3), which is based on maximum hydrogen producing ability of the internal reforming reactions.

Consider the demanded fuel cell current to be \( i_{fc,d} \). Then from Eq. (7), the corresponding fuel demand \( \dot{N}_{f,d} \) that satisfies a target \( U_{ss} \) is

\[
\dot{N}_{f,d} = \frac{i_{fc,d}N_{cell}}{4nFU_{ss}} \left[ 1 - \left( 1 - U_{ss} \right) k \right]
\]

Eq. (8) only addresses steady-state behavior. Hence we must assess its effectiveness in the
presence of transient current demand. Control of $U$ using Eq.(8) is shown through sample simulations in Fig.2. The SOFC system model, with cell length of 50cm and cell area of 251cm$^2$, is simulated with $N_{cell} = 50$ and with $i_{fc} = 10A$ for $t < 150s$ and target $U_{ss} = 85\%$. As mentioned in the Introduction, such high fuel utilization is desirable from the cell’s electric efficiency standpoint. Four simulations are presented with $i_{fc} = 11, 14, 18, 22A$ for $t \geq 150s$, Fig.2(a). The actual fuel injected, $\dot{N}_{f}$, is shown in Fig.2(b). Note that while $\dot{N}_{f,d}$ changes instantaneously according to Eq.(8), $\dot{N}_{f}$ experiences a lag. This is due to the lag introduced by the fuel supply system consisting of fuel pump and/or valves, to be discussed further in sections 4 and 5. At steady-state, $\dot{N}_{f} = \dot{N}_{f,d}$. In this simulation, we have assumed a first order dynamics with a time-constant of 2s. However, similar response is obtained with other fuel supply dynamics, such as ramped or rate-limited behavior. In Fig.2(c), the plots confirm $U_{ss} = 85\%$. The results show high sensitivity of transient $U$ to load fluctuations in open-loop operation. $V_{fc}$ is plotted in Fig.2(d). The simulation abruptly ended when the step increase was $> 10A$. This is due to hydrogen starvation, manifested by $U \rightarrow 100\%$ and $V_{fc} \rightarrow 0$, as seen in Figs.2(c) and (d).

From Fig.2 we make the following observations. Under open-loop control, the SOFC is capable of handling limited amount of current fluctuation with $U$ deviating from target $U_{ss}$ during transients. This deviation is a result of the time delay between a change in the fuel command $\dot{N}_{f,d}$ and its corresponding effect in the fuel cell. Also, $V_{fc}$ experiences overshoot or undershoot during transient.
4 CURRENT REGULATION

The advantage of the approach in section 3 is that target $U_{ss}$ is achieved without the knowledge of internal flow rates, temperatures, species concentrations or reaction rates. However, it leads to transient deviations in $U$. We address this issue by dynamically shaping $i_{fc}$ using feedback. Noting that $\dot{N}_f \neq \dot{N}_{f,d}$ during transients, the fuel cell current $i_{fc}$ is shaped using Eq.(7), as follows:

$$i_{fc} = \frac{4nF U_{ss}\dot{N}_f}{N_{cell}} \frac{1}{[1 - (1 - U_{ss}) k]}$$  \hspace{2cm} (9)

Implementing Eq.(9) requires the measurement of the actual fuel flow $\dot{N}_f$, which is assumed to be available. The feedback based current regulation scheme and the open loop approach are shown in Fig.3 by the switch positions $CR$ and $OL$ respectively. Note that $\dot{N}_f$ is measured upstream of the reformer where the flow consists of pure methane. The reformed flow downstream of the reformer consists of a mixture of $CH_4$, $CO$, $CO_2$, $H_2$ and $H_2O$, whose composition and flow rate are assumed to be unknown. The proposed method has similarities with existing model-based load governors proposed in [9, 38] for PEMFC and SOFC-GT systems. Another related approach appears in [39] where a filter is added to the fuel cell power request based on the characteristics of the air delivery system. In comparison, using the sensed fuel in conjunction with the invariant relationship in Eq.(9) obviates the need for characterization of the fuel supply system (FSS), Fig.3. Although considered feedback based regulation, note that with respect to the fuel cell stack and power electronics subsystem of Fig.3, the proposed method of regulating $i_{fc}$ would be a scheduled feed-forward.

Figure 3: Scheme for Transient Utilization Control
Simulations are presented in Fig.4 to demonstrate the effect of current regulation on transient utilization and voltage. The system simulated is the same as that in Fig.2. Referring to Fig.3, the simulation results represent the current regulation (CR) mode. Three simulations are presented, with target $U_{ss} = 85\%$, $i_{fc} = 10A$ for $t < 150s$, and $i_{fc,d} = 18, 30, 50A$ for $t \geq 150s$. In response to the step changes in $i_{fc,d}$, the target fuel $\dot{N}_{f,d}$ also undergoes step changes, Figs.4(a) and (b). $\dot{N}_{f}$ changes according to the fuel supply dynamics which is assumed to be first order with a time constant of 2s, Fig.4(b). Measured $\dot{N}_{f}$ is assumed to be available and the shaped $i_{fc}$ is computed using Eq.(9) and is shown in Fig.4(a). The resulting transient $U$ and $V_{fc}$ are shown in Figs.4(c) and (d).

5 DELAYS INDUCED ALONG FUEL PATH

It is evident from the results above that regulation of $i_{fc}$ drastically reduces transient $U$ and attenuates overshoot and undershoot in $V_{fc}$. For instance, the step change to $i_{fc,d} = 18A$ led to a deviation in $U$ up to $\approx 94\%$ in the open-loop mode (Fig.2(c)), and only $\approx 86\%$ in closed-loop mode (Fig.4(c)). Current regulation also increases the fuel cell’s transient current handling capability considerably. Even with a step change to $i_{fc,d} = 50A$, transient $U$ remains within a $\pm 5\%$ range. We next discuss the physical reasons for transient deviation in $U$.

5 DELAYS INDUCED ALONG FUEL PATH

At the end of section 3, we observed that the transient deviation of $U$ from target $U_{ss}$ is due to delays induced along the fuel path. We attribute the delays to two primary factors, D1: The lag between $\dot{N}_{f,d}$ and $\dot{N}_{f}$ introduced by the fuel supply system (FSS), and
**D2:** The dynamics of fuel processor (reformer) and anode.

Current regulation in section 4 compensates for D1. The residual transient in $U$ shown in Fig.4(c) is attributed to D2. In sections 3 and 4 we assumed a first order dynamics of the FSS. However, the observations made above are applicable to a wide variety of dynamic responses. As an example, we consider a ramped response of the FSS in Fig.5. For $t < 150$s, $i_{fc} = 10$A. Two simulations are shown, with $i_{fc,d} = 20$ and 30A for $t \geq 150$s. $\dot{N}_f$ ramps at a rate of 0.002moles/s in response to change in $i_{fc,d}$, Fig.5(b). As shown in Fig.5(a), $i_{fc} = i_{fc,d}$ in OL and is ramped in CR. $U$ in OL and CR configurations are shown in Figs.5(c) and (d) respectively. As before, we observe significant deviation in $U$ in OL mode that is considerably attenuated in CR mode.

As the delay D1 becomes smaller, the effect of current regulation diminishes. This trend is shown in Fig.6 where for $t < 150$s, $i_{fc} = 10$A and $i_{fc,d} = 15$A for $t \geq 150$s. Three simulations are shown, all with first order fuel-supply dynamics and time-constants chosen as 0.5s, 2s and 6s. It is clear from Fig.6 that greater the delay D1, greater is the degradation of the transient response of $U$ in OL mode, and greater is the effect of current regulation. For perfect disturbance rejection, $i_{fc}$ must be further regulated to compensate for delay D2. The effect D1 is more pronounced than D2 in majority of cases. However, the effect of D2 is magnified when the reformer’s void volume is much larger compared to the anode volume or when there exists severe flow restriction between the reformer and the anode. The latter effect is demonstrated in Fig.7, where the corresponding friction factor is increased by a
Figure 6: Effect of D1 on Transient $U$ in OL and CR Modes

Figure 7: Effect of D2 on Transient $U$ in OL and CR Modes

factor of 100 in the model. In the presence of heightened flow restriction, the transient deviation in $U$ is not as well attenuated as in the default case.

6 HYBRID FUEL CELL CONFIGURATION

Regulation of $i_{fc}$ leads to a mismatch between the demanded power and the fuel cell delivered power. This mismatch is compensated by augmenting the fuel cell with an ultra-capacitor. The hybrid system architecture used in this research is shown in Fig.8. The architecture has similarities to that adopted in [10]. Alternate approaches for interfacing fuel cell and ultra-capacitor/battery are discussed in [4, 6, 40] and references therein. In Fig.8, the fuel cell and the ultra-capacitor are connected to the electrical bus through DC/DC converters, $C_1$ and $C_2$. In the ensuing control development, we treat the DC/DC converters as static power conversion devices. From Fig.8, we have the instantaneous power balance equation

$$V_Li_L = \eta_1 V_{fc}i_{fc} + \eta_2 V_{uc}i_{uc}$$ (10)
7 CONTROL DESIGN

7.1 Control Objectives

Based on the system described in section 6 the control objectives are,

- supply the net power demand $V_L i_L$ at any instant,
- attenuate fluctuation of $U$ through current regulation, as per section 4, and
- maintain the $SOC$ of the ultra-capacitor at a target value,

under the influence of power fluctuations and in the presence of system uncertainties. Control is designed with the following considerations:

1. The bus (supply) voltage $V_L$ is held constant. This can be implemented by operating $C_1$ in voltage control mode and $C_2$ in the current control mode or vice-versa. Without any loss of generality, we follow the former combination with $C_2$ following the commanded ultra-capacitor current $i_{uc,c}$ (we treat $i_{uc} = i_{uc,c}$).

2. The ultra-capacitor current $i_{uc}$ and the fuel demand $\dot{N}_{f,d}$ are treated as control inputs.

3. Measurements of $V_{fc}, V_{uc}, i_L, i_{fc}$ and $\dot{N}_f$ are available.
4. The DC/DC converter efficiencies $\eta_1$ and $\eta_2$ are unknown and time varying with known upper and lower bounds. The bounds are expressed as follows

$$0 < \eta_{1,\text{min}} \leq \eta_1(t) \leq \eta_{1,\text{max}}, \quad 0 < \eta_{2,\text{min}} \leq \eta_2(t) \leq \eta_{2,\text{max}}$$

(11)

The controller uses the upper and lower bounds given in Eq.(11), and nominal values $\bar{\eta}_1 \in [\eta_{1,\text{min}}, \eta_{1,\text{max}}]$ and $\bar{\eta}_2 \in [\eta_{2,\text{min}}, \eta_{2,\text{max}}]$.

### 7.2 Nonlinear Control

We now detail a nonlinear control strategy given schematically in Fig.9. The strategy incorporates fuel cell current regulation shown in Fig.3. However in comparison, Fig.9 uses $i_{uc}$ as a control input instead of $i_{fc}$. This is due to our specific hardware configuration, as mentioned in condition 1 above. $\dot{N}_{f,d}$ is the second input. Let the fuel cell serve as the primary energy source with the ultra-capacitor supplying transient demands. Additionally, if $\eta_1$ was known, then at any instant the fuel cell current demand $i_{fc,d} = V_L \dot{i}_L / \eta_1 V_{fc}$. However, for controlling the ultra-capacitor’s SOC under the uncertainty in $\eta_1$, we design

$$i_{fc,d} = \frac{V_L \dot{i}_L}{\bar{\eta}_1 V_{fc}} + g(E_s) + \delta_1, \quad E_s = S - S_t, \quad S = \frac{V_{uc}}{V_{\text{max}}}$$

(12)
where $S$ is the instantaneous SOC of the ultra-capacitor, $S_t$ is the target SOC, and $V_{max}$ is the maximum ultra-capacitor voltage. The function $g(E_s)$ and the robustness term $\delta_1$ will be designed in the ensuing analysis. The fuel demand $\dot{N}_{f,d}$, is an algebraic function of $i_{fc,d}$, given by Eq.(8), that satisfies the desired $U_{ss}$. The target fuel cell current $i_{fc,t}$, is based on $\dot{N}_f$ as shown in Fig.9, and is computed using Eq.(9), expressed as

$$i_{fc,t} = \frac{4nFU_{ss}\dot{N}_f}{N_{cell}} \frac{1}{[1 - (1 - U_{ss})k]}$$

(13)

The current $i_{fc,t}$ in Eq.(13) must be achieved to obtain the steady-state utilization $U_{ss}$. In the presence of uncertainties, it is achieved through the ultra-capacitor current command,

$$i_{uc} = i_{uc,c} = \frac{V_Li_{L} - \eta_1V_{fc}i_{fc,t}}{\eta_2V_{uc}} + h(E_{fc}) + \delta_2, \quad E_{fc} = i_{fc} - i_{fc,t},$$

(14)

where the function $h(E_{fc})$ and the robustness term $\delta_2$ will be designed in the ensuing analysis.

Our control objectives are to stabilize the origin $E_s = E_{fc} = 0$ in the presence of $i_L$ with the designs of $i_{fc,d}$ and $i_{uc}$ in Eqs.(12) and (14) respectively. We now state the following assumptions about the fuel supply system (FSS):

- The FSS is assumed to be a combination of components such as a fuel pump and/or valves and a controller. The closed-loop system delivers flow $\dot{N}_f$ in response to $\dot{N}_{f,d}$.
- The dynamic equation of $\dot{N}_f$, $\ddot{N}_f = f(\dot{N}_f, \dot{N}_{f,d}, t)$, is assumed to be unknown.
- $\dot{N}_f$ tracks the reference signal $\dot{N}_{f,d}$ such that the error $E_{fl} = \dot{N}_f - \dot{N}_{f,d}$, satisfies a general characteristic. In Theorem 1 we assume $|E_{fl}|$ to decay exponentially, which is relaxed to boundedness of $E_{fl}$ in Theorem 2.

We state and prove the following theorems:

**Theorem 1** If $E_{fl}(t)$ satisfies a general exponential decay condition given by

$$|E_{fl}(t)| \leq \gamma |E_{fl}(t_0)| e^{-\zeta(t-t_0)}, \quad \forall \quad |E_{fl}(t_0)| < r_0$$

(15)
where \(\gamma, \zeta, r_0 > 0\) are constants, then the hybrid SOFC system in Fig.8, with the control approach in Fig.9, satisfies exponential stability of the origin of \(E = [E_s \ E_f c, t \ E_f c]^T\), \(E_{f c, t} = i_{f c, t} - i_{f c, d}\), with \(i_{f c, d}\) and \(i_{u c}\) as in Eqs.(12) and (14) respectively,

\[
g(E_s) = -k_s E_s, \quad k_s > 0, \quad h(E_{f c}) = k_p E_{f c} + k_d \dot{E}_{f c}, \quad k_p, k_d > 0, \quad (16)
\]

and \(\delta_1\) and \(\delta_2\) are chosen to satisfy

\[
\delta_1 \begin{cases} 
< (V_l i_l / V_{f c}) (1/\eta_{1,\max} - 1/\bar{\eta}_1) & \text{for } E_s > 0 \\
> (V_l i_l / V_{f c}) (1/\eta_{1,\min} - 1/\bar{\eta}_1) & \text{for } E_s \leq 0 
\end{cases}
\]

\[
\delta_2 \begin{cases} 
\geq \beta_u \quad \text{for } E_{f c} > 0 \quad \beta_u = \frac{1}{V_{u c}} \left[ V_l i_l \left( \frac{1}{\eta_{1,\min}} - \frac{1}{\bar{\eta}_1} \right) - V_{f c} i_{f c} \left( \frac{\eta_{1,\min}}{\eta_{2,\max} \eta_1} - \frac{\eta_1}{\bar{\eta}_2} \right) \right] \\
\leq \beta_l \quad \text{for } E_{f c} \leq 0 \quad \beta_l = \frac{1}{V_{u c}} \left[ V_l i_l \left( \frac{1}{\eta_{1,\max}} - \frac{1}{\bar{\eta}_1} \right) - V_{f c} i_{f c} \left( \frac{\eta_{1,\max}}{\eta_{2,\min} \eta_1} - \frac{\eta_1}{\bar{\eta}_2} \right) \right] 
\end{cases} \quad (18)
\]

**Proof:** From Eqs.(8), (13) and Fig.9, we have

\[
\dot{\bar{N}}_{f,d} = \sigma i_{f c,d}, \quad \dot{\bar{N}}_f = \sigma i_{f c,t}, \quad \sigma = \frac{N_{cell}}{4 n F U_{s s}} \left[ 1 - (1 - U_{s s}) k \right], \quad \Rightarrow E_{f l} = \sigma E_{f c,t} \quad (19)
\]

where \(\sigma\) is constant. Eqs.(15) and (19) imply

\[
|E_{f c,t}(t)| \leq \gamma |E_{f c,t}(t_0)| e^{-\zeta(t-t_0)}, \quad \forall \quad |E_{f c,t}(t_0)| < r_0 / \sigma \quad (20)
\]

We observe from Eq.(20) and *Converse Lyapunov Theorems* [17], that there exists a positive definite function \(\bar{V}_{f c}\) and constants \(\alpha_1, \alpha_2, \alpha_3\), such that,

\[
\alpha_1 E_{f c,t}^2 \leq \bar{V}_{f c}(E_{f c,t}, t) \leq \alpha_2 E_{f c,t}^2, \quad 0 < \alpha_1 < \alpha_2 \quad \text{and} \quad \dot{\bar{V}}_{f c} \leq -\alpha_3 E_{f c,t}^2, \quad \alpha_3 > 0 \quad (21)
\]
We choose the Lyapunov function candidate $V = 0.5 \left( E_s^2 + E_{fc}^2 \right) + \bar{V}_{fc}$ and note that it is positive definite and decreasing with

$$\min(\alpha_1, 0.5) \| \mathcal{E} \|^2 \leq \dot{V} = \frac{1}{2} \left( E_s^2 + E_{fc}^2 \right) + \bar{V}_{fc} \leq \max(\alpha_2, 0.5) \| \mathcal{E} \|^2, \quad \mathcal{E} = \begin{bmatrix} E_s & E_{fc,t} & E_{fc} \end{bmatrix}^T$$ (22)

Next, from the dynamical equation of the ultra-capacitor and using Eq.(12) we have,

$$\dot{V}_{uc} = -i_{uc}/C \Rightarrow \dot{E}_s = -i_{uc} / (C V_{max})$$ (23)

From Eqs.(10), (12), (14), (16) and (23), noting that

$$E_{fc} = i_{fc} - i_{fc,t}, \ E_{fc,t} = i_{fc,t} - i_{fc,d} \Rightarrow i_{fc} = E_{fc} + E_{fc,t} + i_{fc,d}$$

we have,

$$\dot{E}_s = -(1/CV_{max}) \left[ (V_L i_L / \eta_2 V_{uc}) - (\eta_1 V_{fc} / \eta_2 V_{uc}) \left\{ E_{fc} + E_{fc,t} + (V_L i_L / \bar{\eta}_1 V_{fc}) - k_s E_s + \delta_1 \right\} \right]$$ (24)

Next, note from Eqs.(10), (14) and (16) that

$$V_L i_L = \eta_1 V_{fc} i_{fc} + \eta_2 V_{uc} \left[ \frac{V_L i_L - \bar{\eta}_1 V_{fc} i_{fc,t}}{\bar{\eta}_2 V_{uc}} + k_p E_{fc} + k_d \dot{E}_{fc} + \delta_2 \right]$$ (25)

Rearranging Eq.(25) we have

$$\dot{E}_{fc} = -\alpha E_{fc} + \frac{\beta - \delta_2}{k_d}$$ (26)

where,

$$\alpha = [k_p / k_d + (V_{fc} / k_d V_{uc}) (\bar{\eta}_1 / \bar{\eta}_2)] > 0, \quad \beta = \frac{1}{V_{uc}} \left[ V_L i_L \left( \frac{1}{\eta_2} - \frac{1}{\bar{\eta}_2} \right) - V_{fc} i_{fc} \left( \frac{\eta_1}{\eta_2} - \frac{\bar{\eta}_1}{\bar{\eta}_2} \right) \right]$$ (27)
From Eqs. (21), (22), (24) and (26), we have,

\[
\dot{\bar{V}} \leq -\mathcal{E}^T Q \mathcal{E} + \frac{E_s \eta_1 V_{fc}}{CV_{max} \eta_2 V_{uc}} \left[ \delta_1 - \frac{V_L i_L}{V_{fc}} \left( \frac{1}{\eta_1} - \frac{1}{\bar{\eta}_1} \right) \right] + \frac{E_{fc}}{k_d} (\beta - \delta_2) \tag{28}
\]

\[
\mathcal{E} = \begin{bmatrix} E_s \\ E_{fc,t} \\ E_{fc} \end{bmatrix}, \quad Q = m \begin{bmatrix} k_s & -0.5 & -0.5 \\ -0.5 & \alpha_3/m & 0 \\ -0.5 & 0 & \alpha/m \end{bmatrix}, \quad m = \frac{\eta_1 V_{fc}}{CV_{max} \eta_2 V_{uc}} \tag{29}
\]

Note in Eq. (29) that \( Q \) is symmetric. Furthermore, \( m > 0 \) and has finite positive upper and lower bounds over the range of operation of the hybrid system. Note that \( \alpha_3 \) is not a tunable parameter since it is determined by the dynamics of the FSS. However, by choosing \( k_s \) and \( \alpha \) appropriately, we can ensure \( Q > 0 \, \forall \, m \). Thus, from the Rayleigh-Ritz Inequality [41]

\[
\mathcal{E}^T Q \mathcal{E} \geq \inf (\lambda_{min,Q}) ||\mathcal{E}||^2 > 0 \quad \forall \, \mathcal{E} \neq 0 \tag{30}
\]

where \( \lambda_{min,Q} \) represents the smallest eigenvalue of \( Q \) at any instant. Furthermore, by choosing \( \delta_1 \) and \( \delta_2 \) to satisfy Eqs. (17) and (18), we have from Eqs. (28) and (30)

\[
\dot{V} \leq -\mathcal{E}^T Q \mathcal{E} \leq -\inf (\lambda_{min,Q}) ||\mathcal{E}||^2 < 0, \quad \forall \, \mathcal{E} \neq 0 \tag{31}
\]

The conditions in Eqs. (17) and (18) are implementable with the available voltage and current measurements and known upper and lower bounds of \( \eta_1 \) and \( \eta_2 \). From Eqs. (21), (22) and (31), we conclude that the control design in Eqs. (12) and (14), with functions \( g(E_s) \), \( h(E_{fc}) \) designed as in Eq. (16), and \( \delta_1 \), \( \delta_2 \) chosen as in Eqs. (17) and (18) respectively, guarantee exponential stability of \( \mathcal{E} = 0 \). This completes the proof (see Theorem 4.10 of [17]). ☠️ ☠️ ☠️

**Theorem 2** If \( E_{fl} \) is bounded, then the system in Eqs. (24) and (26) with state vector \( \bar{\mathcal{E}} = [E_s \ E_{fc}]^T \), input \( E_{fc,t} \), and \( \delta_1 \) and \( \delta_2 \) designed as in Eqs. (17) and (18) respectively, is Input-to-State Stable, ISS [17].
**Proof:** From Eq.(19), boundedness of $E_{fl}$ implies boundedness of $E_{fc,t}$. Choosing the following positive definite and decremental Lyapunov function candidate

$$
\bar{V} = \frac{(E_s^2 + E_{fc}^2)}{2}
$$

we have

$$
\dot{\bar{V}} \leq -\bar{\xi}^T \bar{Q} \bar{\xi} + \frac{E_s \eta_1 V_{fc}}{CV_{max} \eta_2 V_{uc}} \left[ (\delta_1 + E_{fc,t}) - \frac{V_{j_x}}{V_{fc}} \left( \frac{1}{\eta_1} - \frac{1}{m} \right) \right] + \frac{E_{fc}}{k_d} (\beta - \delta_2)
$$

$$
\bar{Q} = m \begin{bmatrix}
k_s & -0.5 \\
-0.5 & \alpha/m
\end{bmatrix}, \quad m = \eta_1 V_{fc}/(CV_{max} \eta_2 V_{uc}) > 0
$$

where $\alpha$ and $\beta$ are defined in Eq.(27). By choosing $k_s$ and $\alpha$ appropriately, we can ensure $\bar{Q} > 0$ for all feasible $m$. Furthermore, by choosing $\delta_1$ and $\delta_2$ to satisfy Eqs.(17) and (18) respectively, we have from Eq.(33),

$$
\dot{\bar{V}} \leq -\bar{\xi}^T \bar{Q} \bar{\xi} + \frac{E_s \eta_1 V_{fc}}{CV_{max} \eta_2 V_{uc}} E_s E_{fc,t} \\
\leq -\inf (\lambda_{min,Q}) \bar{\xi}^2 + (\eta_1 V_{fc}/CV_{max} \eta_2 V_{uc}) ||\bar{\xi}|| \left| E_{fc,t}(t) \right| \\
\leq -\inf (\lambda_{min,Q}) (1 - \theta) ||\bar{\xi}||^2, \quad 0 < \theta < 1 \\
< 0, \quad \forall \quad ||\bar{\xi}|| \geq \eta_1 V_{fc}/[CV_{max} \eta_2 V_{uc} \theta \inf (\lambda_{min,Q})] \left| E_{fc,t}(t) \right|
$$

The conditions of Theorem 4.19 of [17] are satisfied and we conclude the system is ISS. ◇◇◇

The system in Eqs.(24) and (26) is obtained with $i_{fc,d}$ and $i_{uc}$ as in Eqs.(12) and (14), and functions $g(E_s)$ and $h(E_{fc})$ designed as in Eq.(16). The bound on $\bar{\xi}$ will depend on the bound on $E_{fc,t}$. Hence, the more accurately $\dot{N}_f$ tracks $\dot{N}_{f,d}$, lower will be the bound on $\bar{\xi}$. The boundedness condition of Theorem 2 is less restrictive and more realistic than exponential decay considered in Theorem 1.
7.3 $H_\infty$ Control

The nonlinear control discussed in section 7.2 leads to a custom algorithm that addresses system uncertainties. We also explore the application of standard robust control techniques for this problem. The advantage of standard methods lies in the availability of computational tools for controller synthesis. Specifically, we show that the simplicity of the ultra-capacitor’s dynamic model, Eq.(23), allows the design of an $H_\infty$ control for SOC regulation. Referring to Fig.9 and Eq.(12), the only difference introduced by the $H_\infty$ approach compared to the nonlinear control in section 7.2 is that $i_{f,c,d}$ is computed as

$$i_{f,c,d} = V_L i_L / \bar{\eta}_1 V_{fc} + \delta_{i_{f,c}}$$

where, $\delta_{i_{f,c}}$ is obtained through an $H_\infty$ synthesis. From Eqs.(10) and (23), we have

$$\dot{V}_{uc} = [V_L i_L - \eta_1 V_{fc} i_{f,c}] / CV_{uc} \eta_2$$

Expressing $i_{f,c}$ as $i_{f,c} = i_{f,c,d} + \epsilon$, where $\epsilon$ represents a bounded unknown error term, we get from Eqs.(35) and (36)

$$\dot{V}_{uc} = f - p(\delta_{i_{f,c}} + \epsilon), \quad f = V_L i_L \left( 1 - \frac{\eta_1}{\bar{\eta}_1} \right) / CV_{uc} \eta_2, \quad p = V_{fc} \eta_1 / CV_{uc} \eta_2$$

We design $\delta_{i_{f,c}}$ as follows,

$$\delta_{i_{f,c}} = \frac{1}{\hat{p}} \left( \hat{f} - v \right) \Rightarrow i_{f,c,d} = \frac{V_L i_L}{\bar{\eta}_1 V_{fc}} + \frac{1}{\hat{p}} \left( \hat{f} - v \right)$$

where $\hat{f}$ and $\hat{p}$ are the estimates of the functions $f$ and $p$ respectively, evaluated with nominal values. Substituting for $\delta_{i_{f,c}}$ from Eq.(38) into Eq.(37), we get

$$\dot{V}_{uc} = \frac{p}{\hat{p}} v + \left[ f - \frac{p}{\hat{p}} \hat{f} - p \epsilon \right]$$
Representing $p/\dot{p}$ as a nominal value $\beta_0$ plus deviation $\Delta \beta_0$, and assigning the remaining term on the right hand side of Eq.(39) as disturbance $d$, we have

$$\dot{V}_{uc} = (\beta_0 + \Delta \beta)v + d$$

where, the modified control input $v$ is computed using the dynamic feedback law

$$v(s) = -K_\infty(s)e(s), \quad e(t) = V_{uc,t} - V_{uc}$$

where $K_\infty$ is controller transfer function obtained using standard $H_\infty$ synthesis [42], and $V_{uc,t} = S_t V_{max}$ is the target ultra-capacitor voltage. The $H_\infty$ synthesis is carried out using MATLAB®’s robust control toolbox. We end this section with the following remark:

**Remark 1** Throughout our control design, we assumed $C_1$ and $C_2$ to operate in voltage and current control modes respectively. If the combination is reversed, $E_{fc} = 0$, i.e. $i_{fc} = i_{fc,t}$, and current control of the ultra-capacitor, and hence Eq.(14), are irrelevant. Referring to Theorems 1 and 2, exponential decay of $|E_{fl}|$ and boundedness of $E_{fl}$ would still imply exponentially stable and ISS property of $E = 0$ and $\bar{E}$ respectively with $i_{fc,d}, E_s$ and $\delta_1$ designed as in Eqs.(12), (16) and (17). The $H_\infty$ control design would remain unchanged.

The proof of the above remark is omitted for brevity. Finally, note that although stack temperature control is important, it is not considered in our control development. This is because stack temperature transients are considerably slower (order of tens of minutes) compared to transient $U$ (order of tens of seconds) [34], [7]. Also, in absence of coolants, SOFC stack temperature is typically controlled separately by manipulating the cathode air.

**8 EXPERIMENTAL TEST-STAND**

We next test the control strategies developed in sections 7.2 and 7.3 on an experimental test-stand, shown in Fig.10 which implements the hybrid system schematically depicted in
Fig. 8. The test-stand consists of the following:

- **Fuel Cell Emulator**: An SOFC system is emulated by executing its mathematical model on a dSPACE® DS1103 real time processor in conjunction with a 100V/50A programmable power supply. A schematic diagram of the emulator is shown in Fig.11. The DS1103PPC controller is fully programmable from the MATLAB®/Simulink® environment. The power supply is run in voltage control mode with isolated analog input from the DS1103PPC controller.

- **Electronic Load**: A DC electronic load, shown in Fig.10, is used for power consumption. The electronic load can draw a maximum power of 1.8kW with a maximum voltage of 60V and a maximum current of 120A.

- **Unidirectional DC/DC Converter**: A unidirectional DC/DC converter, denoted by $C_1$ in Fig 11, maintains $V_L = 24V$ at the output, with output current rating of $\approx 33A$. 
The efficiency of the converter varies between 85% and 93%.

- **Voltage Measurement**: The voltage of the ultra capacitor is measured using voltage probes with high input impedance, as shown in Fig.10.

- **The Ultra-capacitor Module**: A 16V series BMOD0250-E016 ultra capacitor from MAXWELL Technologies is used in this experimental setup. The module has specifications of \( C = 250 \text{F} \), \( V_{max} = 16.2 \text{V} \), and an internal resistance of \( \approx 4.1 \text{m\Omega} \).

- **Bidirectional DC/DC Converter**: A bidirectional DC/DC converter, denoted by \( C_2 \) in Fig.11, is used to command the ultra-capacitor current \( i_{uc} \) through analog input.

- **Current Clampers**: Current measurement is obtained using two Fluke 80i-110s AC/DC current probes, used for measuring \( i_L \) and \( i_{fc} \).

9 EXPERIMENTAL RESULTS

In this section, we present results of experiments carried out on the test-stand described in section 8. We first present the results of the nonlinear control strategy followed by those of the \( H_{\infty} \) approach. In all tests, we emulate an SOFC system with 50 cells connected in series. In the plant model, the fuel supply dynamics is modeled to follow a first order dynamics with \( \dot{N}_f(s) = [1/(2s + 1)] \dot{N}_{f,d}(s) \). Neither control designs assume a knowledge of this dynamics. The bus voltage is maintained at \( V_L = 24 \text{V} \) by converter \( C_1 \) in all experiments.
### 9.1 Nonlinear Control

We present results of two experiments where the nonlinear control strategy of section 9 was applied. The results are presented in Figs.12 and 13. Both experiments use the following parameter values: $U_{ss} = 0.8$, $S_t = 0.8$, $\bar{\eta}_1 = 1$, $\bar{\eta}_2 = 0.98$, $k_s = 0.1$ and $k_p = 0.2$. The parameters $\delta_1$ and $\delta_2$ were chosen based on Eqs.(17) and (18), as follows:

$$
\begin{align*}
\delta_1 \begin{cases}
0 & \text{for } E_s > 0 \\
\bar{\delta}_1 & \text{for } E_s \leq 0
\end{cases}, \\
\delta_2 \begin{cases}
2 & \text{for } E_{fc} > 0 \\
-2 & \text{for } E_{fc} \leq 0
\end{cases}
\end{align*}
$$

(42)

In Fig.12, $i_L$ follows the current profile of Fig.12(a). This profile represents a standard drive cycle obtained from [43], where the velocity variation was simply interpreted as current variation. For this test we set $\bar{\delta}_1 = 4$ and $k_d = 0.0004$. The variables related to the fuel cell, $i_{fc}$, $V_{fc}$, $U$ and $\dot{N}_f$ are plotted in Figs.12(b), (c), (d) and (e) respectively. The results show
very close control of $U$ in spite of rapidly varying $i_L$. The variables $S$ and $i_{uc}$ are plotted in Figs.12(f) and (g) respectively. The plots show tight control of $S$ around the target $S_t = 0.8$. Fuel cell current $i_{fc}$ is consistently lower than $i_L$ since $V_{fc}$ is higher than the bus voltage $V_L = 24V$. The rapid variation of $i_{uc}$ around zero shows that the ultra-capacitor supplies transient power only. The ultra-capacitor current $i_{uc}$ is magnified for $600 \leq t \leq 800$ to show its variation in detail. The fuel cell and ultra-capacitor powers are plotted in Fig.12(h).

In Fig.13(a), $i_L$ is subject to step changes. For this test we set $\delta_1 = 2$ and $k_d = 0.004$. The corresponding variations in $i_{fc}$, $V_{fc}$, $U$ and $N_f$ are plotted in Figs.13(b), (c), (d) and (e) respectively. In Fig.13(b), both $i_{fc}$ and $i_{fc,t}$ are plotted together but the difference is not clear as they are almost coincident. $S$ and $i_{uc}$ are plotted in Figs.13(f) and (g) respectively, and the fuel cell and ultra-capacitor power are plotted in Fig.13(h). The results show very close control of $U$ in spite of drastic transients in $i_L$. In comparison to Figs.12(f), (g) and (h), Figs.13(f), (g) and (h), show greater variations in $S$, $i_{uc}$ and the ultra-capacitor instantaneous power. This is because step changes introduce faster transients than the drive cycle.
### 9.2 $H_\infty$ Control

The results of $H_\infty$ control are presented in Fig.14. In this experiment, the following control parameters were chosen: $U_{ss} = 0.8$, $S_t = 0.8$, $k_p = 0.2$, and $k_d = 0.004$. The following ranges were chosen for uncertain quantities with nominal values at the middle of their respective ranges, $\bar{\eta}_1, \bar{\eta}_2 \in [0.8 \, 1]$, $V_{fc} \in [40 \, 60] \text{V}$ and $V_{uc} \in [9.6 \, 16.2] \text{V}$. Referring to Eq.(41), the following dynamic feedback law was determined through $H_\infty$ synthesis,

$$K_\infty = -\frac{v(s)}{e(s)} = \frac{3.587 \times 10^4 s + 1.75 \times 10^4}{s^2 + 1049s + 105.4}$$

In Fig.14, $i_L$ is subject to the same step changes as in Fig.13. Figs.13 and 14 indicate comparable performance of the $H_\infty$ controller and the nonlinear control strategy. The only noticeable difference is in the SOC control, Figs.13(f) and 14(f), where the nonlinear control seems to perform better. While the $H_\infty$ design was simplified due to the existence of numerical tools (MATLAB®’s robust control toolbox), it took considerable tuning effort to obtain acceptable performance. On the other hand, while controller development was more involved for the nonlinear control, tuning efforts were lesser.

![Figure 14: $H_\infty$ Control under Step Changes in $i_L$](image)
10 CONCLUSION

This paper addresses the control of a hybrid SOFC ultra-capacitor system to improve its load-following capability while preventing fuel starvation in the SOFC. The fuel starvation problem is addressed through transient control of fuel utilization, $U$, achieved by regulating the fuel cell current $i_{fc}$ using an invariant relationship involving $U$, fuel flow $\dot{N}_f$ and $i_{fc}$. The approach leads to a mismatch between requested power and fuel cell delivered power during transients. This mismatch is compensated by the ultra-capacitor. For the hybrid system, a comprehensive control strategy is required that incorporates current regulation and additionally controls the ultra-capacitor’s SOC at a target level. To this end, two control designs are proposed. The first is a robust nonlinear control strategy for which stability properties of the closed-loop system are guaranteed. The second is a standard $H_\infty$ approach. To test the control strategies an experimental test-stand is developed, consisting of an emulated SOFC, and actual ultra-capacitor and power-electronics components, forming a laboratory scale hybrid power system. Both control strategies show comparable performance on this platform. In future research, conditions such as saturated fuel flow, limiting current density of fuel cell, limiting ultra-capacitor current, which are more likely to occur at higher power applications, will be considered. In such cases, addition of a battery is foreseen.

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